Example 1

1. *A large volcanic eruption occurs and the resulting SO_2 in the lower stratosphere is converted to sulfate aerosols. What are the qualitative radiative effects?*

   The eruption of Mount Pinatubo in 1991 was measured by ERBE to increase the global albedo by 0.02 and decrease the longwave flux by 3.3 W/m^2. What was the total radiative forcing?

   Suppose the climate sensitivity is 0.6 $K/(W/m^2)$. What is the resulting global average surface temperature change?

The major radiative effect is to increase the shortwave reflectivity of the Earth, which is a negative radiative forcing (cooling). The sulfate aerosols also absorb infrared radiation so less longwave radiation is emitted to space, which is a positive radiative forcing (warming).

Our definition of radiative forcing is the instantaneous change in the top of the atmosphere net radiative flux:

$$\Delta Q = -\Delta\bar{r}(S/4) - \Delta F_{LW}$$

Therefore, for this eruption

$$\Delta Q = -0.02(1366/4) + 3.3 = -6.8 + 3.3 = -3.5 \text{ W/m}^2$$

The surface temperature change is estimated from the radiative forcing and the climate sensitivity by

$$\Delta T = G\,\Delta Q \;\; = \;\; (-3.5 \text{ W/m}^2)\,(0.6 \text{ K W}^{-1}\text{m}^2) \;\; = -2.1\;K$$

Example 2

2. *Derive the formula for determining the feedback factor for a single feedback in a climate model.*

The feedback factor is defined to be

$$f_j = G_0 \frac{\partial N}{\partial X_j} \frac{\partial X_j}{\partial T}$$

where G_0 is the climate sensitivity with no feedbacks, N is the TOA net radiative flux, X_j is the climate variable of the feedback, and T is the global average surface temperature.

We can't measure just one partial derivative (one feedback) directly in a climate model, because the climate sensitivity is due to the total derivative (direct response and all feedbacks). The temperature difference between the run with radiative forcing and the control run is related to the climate sensitivity G and the total derivative dN/dT by

$$\Delta T_{all} \ = \ G\Delta Q \ = \ -\frac{\Delta Q}{dN/dT}$$

To isolate the one feedback we need to subtract two total derivatives:

$$\left.\frac{\partial N}{\partial T}\right|_j = \left.\frac{dN}{dT}\right|_{\text{all}} - \left.\frac{dN}{dT}\right|_{\text{no } j}$$

Thus the feedback factor is

$$f_j = G_0 \left[\left.\frac{dN}{dT}\right|_{\text{all}} - \left.\frac{dN}{dT}\right|_{\text{no } j} \right] \ = \ G_0 \left[\frac{-\Delta Q}{\Delta T_{\text{all}}} - \frac{-\Delta Q}{\Delta T_{\text{no } j}} \right]$$

Remembering that $\Delta T_0 = G_0 \Delta Q$ we get

$$f_j = \left[\frac{1}{\Delta T_{\text{no } j}} - \frac{1}{\Delta T_{\text{all}}} \right] \Delta T_0$$